DIFFERENTIATION

Derivative

(i) The rate of change of one quantity with respect to some other quantity has a great importance. For example the rate of change of displacement of a particle with respect to time is called its velocity and rate of change of velocity is called its acceleration. The rate of change of a quantity 'y' with respect to another quantity 'x' is called the derivative or differential coefficient of y with respect to x .

(ii) Let y = f(x) be a continuous function of an independent variable x. Let dx be an arbitrary small change in the value of x and dy be the corresponding change in that of y. Then limit of the ratio $\frac{\delta y}{\delta x}$ as $\delta x \to 0$, if exists, is named as the derivative or differential coefficient of x with respect to y and it is denoted by dy / dx

So $\frac{dy}{dx} = \lim_{x \to 0} \frac{\delta y}{\delta x}$

i.e. $\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x} = f'(x)$

Derivative of y with respect to x is also denoted by the following symbols : $y¢$, y_1 , Dy

The process of finding derivative of a function is called differentiation.

Derivatives of some standard functions

Some Differentiation Formulae

(i) $\frac{d}{dx}(\text{constant}) = 0$

(ii) $\frac{d}{dx}(x^n) = nx^{n-1}$

(iii) $\frac{d}{dx}(e^x) = e^x$

(iv) $\frac{d}{dx}(a^x) = a^x \log_e a$

(v) $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

(vi) $\frac{d}{dx}(\log_a x) = \frac{1}{x \log_e a}$

(vii) $\frac{d}{dx}(\sin x) = \cos x$

(viii) $\frac{d}{dx}(\cos x) = -\sin x$

(ix) $\frac{d}{dx}(\tan x) = \sec^2 x$

(x) $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$

(xi) $\frac{d}{dx}(\sec x) = \sec x \tan x$

(xii) $\frac{d}{dx}(\text{cosec } x) = -\text{cosec } x \cot x$

(xiii) $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$; $-1 < x < 1$

(xiv) $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$; $-1 < x < 1$

(xv) $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$; $x \in R$

(xvi) $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$, $\forall \ x \in R$

(xvii) $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$, $|x| > 1$

(xviii) $\frac{d}{dx}(\text{cosec}^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$; $|x| > 1$

(xix) $\frac{d}{dx}(e^{ax}\sin bx) = e^{ax}(a\sin bx + b\cos bx) \quad = \sqrt{a^2+b^2}\; e^{ax}\sin(bx + \tan^{-1}\frac{b}{a})$

(xx) $\frac{d}{dx}(e^{ax}\cos bx) = e^{ax}(a\cos bx - b\sin bx) \quad = \sqrt{a^2+b^2}\; e^{ax}\cos(bx + \tan^{-1}\frac{b}{a})$

(xxi) $\frac{d}{dx}|x| = \frac{x}{|x|}\ (x \neq 0)$ (xxii) $\frac{d}{dx}\log|x| = \frac{1}{x}\ (x \neq 0)$

Some theorems of differentiation

(i) $\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$

(ii) $\frac{d}{dx}(k\,f(x)) = k\frac{d}{dx}(f(x))$, where k is any constant

(iii) $\frac{d}{dx}(f_1(x).f_2(x)) = (f_1(x))\frac{d}{dx}(f_2(x)) + (f_2(x))\frac{d}{dx}(f_1(x))$

(iv) $\frac{d}{dx}\left(\frac{f_1(x)}{f_2(x)}\right) = \frac{f_2(x)\frac{d}{dx}(f_1(x)) - f_1(x)\frac{d}{dx}(f_2(x))}{(f_2(x))^2}$

(v) Derivative of the function of the function .

If 'y' is a function of 't' and 't' is a function of 'x', then

$$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx}$$

Thus $\frac{d}{dx}\{g[f(x)]\} = \frac{dg\{f(x)\}}{df(x)} \cdot \frac{d}{dx}f(x) = g'\{f(x)\}.\ f'(x)$

(vi) Derivative of parametric equation

If $x = \phi(t)$, $y = \psi(t)$ then,

$$\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt}$$

(vii) Derivative of a function w.r..t another function

If f(x) and g(x) are two function of a variable x, then

$$\frac{d(f(x))}{d(g(x))} = \frac{d}{dx}(f(x)) \Big/ \frac{d}{dx}(g(x))$$

Differentiation of Implicit Functions

If in an implicit function f(x, y) = 0, y cannot be expressed in terms of x, then we differentiate both sides of the given equation w.r.t. x and collect all terms containing $\frac{dy}{dx}$ on L.H.S.

NOTE : In the above process we obtain dy/dx in terms of both x and y. If we want dy/dx in terms of x only, then let us first express y in terms of x .

Short method of differentiation for implicit functions

If f(x,y) = constant, then

$$\frac{dy}{dx} = -\left(\frac{\partial f}{\partial x}\right) / \left(\frac{\partial f}{\partial y}\right)$$

where $\left(\frac{\partial f}{\partial x}\right)$ and $\left(\frac{\partial f}{\partial y}\right)$ are partial derivatives of f (x ,y) with respect to x and y respectively.

[By partial derivative of f(x , y) with respect to x, we mean the derivative of f(x, y) with respect to x when y is treated as a constant.]

Logarithmic differentiation

If differentiation of an expression or an equation is done after taking log on both sides, then it is called logarithmic differentiation. Generally we apply this method when given expression is in one of the following forms

(i) $[f(x)]^{g(x)}$

(ii) product of three or more function

Some Suitable substitutions

(i) If the function involve the term $\sqrt{a^2 - x^2}$, then substitute $x = a \sin\theta$ or $x = a \cos\theta$

(ii) If the function involve the term $\sqrt{x^2 + a^2}$, then substitute $x = a \tan\theta$ or $x = a \cot\theta$

(iii) If the function involve the term $\sqrt{x^2 - a^2}$, then substitute $x = a \sec\theta$ or $x = a \operatorname{cosec}\theta$

(iv) If the function involve the term $\left(\sqrt{\frac{a-x}{a+x}}\right)$, then substitute $x = a \cos\theta$ or $x = a \cos 2\theta$

Differentiation of a Determinant Function

If $F(x) = \begin{vmatrix} f & g & h \\ \ell & m & n \\ u & v & w \end{vmatrix}$

Where f, g, h, λ , m, n, u, v, w are functions of x and differentiable , then

$$F'(x) = \begin{vmatrix} f' & g' & h' \\ \ell & m & n \\ u & v & w \end{vmatrix} + \begin{vmatrix} f & g & h \\ \ell' & m' & n' \\ u & v & w \end{vmatrix} + \begin{vmatrix} f & g & h \\ \ell & m & n \\ u' & v' & w' \end{vmatrix}$$

or

$$F'(x) = \begin{vmatrix} f' & g & h \\ \ell' & m & n \\ u' & v & w \end{vmatrix} + \begin{vmatrix} f & g' & h \\ \ell & m' & n \\ u & v' & w \end{vmatrix} + \begin{vmatrix} f & g & h' \\ \ell & m & n' \\ u & v & w' \end{vmatrix}$$